Unit 2 Algebraic Expressions

Monomial : is an algebraic expression with 1 term. It can be:

- *A constant*: 5 ; -3 ; $\frac{1}{2}$
- *A variable*: a ; x ; t
- *A product:* $2a$; $-4x^2$; $3xy$; $\frac{1}{2}x^2y$

Note: the exponent must be a natural number. i.e. $3x^{-2}$; $\frac{3}{x^2}$ $5x^{1/3}$; $2\sqrt{x}$ are not monomials

Coefficient: is the factor by which a variable is multiplied

$3x^n \rightarrow exponent \in \mathrm{N}$

coefficient *variable*

Note: if the coefficient is 1, it is not written, for example: ab = 1ab ; $-1x^2 = -x^2$

Like terms: are terms with identical variables and identical exponents (not coefficients)

Examples: 6 and -2 $-2a^3b^2$ and $5a^3b^2$
3a and 4a $0.5xy^5$ and $10xy^5$

The Degree of a term is the sum of the exponents of the variables.

Examples:				
	3	degree	0	
	3x	degree	1	
	$3x^2$ or 3xy	degree	2	
	$3x^2y$	degree	3	
	$3x^2y^3$	degree	5	etc.

To find the numerical value of an algebraic expression we replace the variable by the given value.

Examples:	$4x^3$ $if x = 2$	$2a^2$ $if a = -3$	$2x^3y^2$ $if x = 2; y = -3$
	$=4(2)^3$	$=2(-3)^2$	$=2(2)^3(-3)^2$
	$=4(8)$	$=2(9)$	$=2(8)(9)$
	$=32$	$=18$	$=144$

Binomial: is an algebraic expression with 2 terms.

Examples: $3x + 2$; $2a^2 + 3a$; $4ab - 2a$

Trinomial: is an algebraic expression with 3 terms.

Examples: $2a^2 + 3a + 5$; $b^3 - 2b + 5$; $2x^2 - 6xy + 7y$

Polynomial: is an algebraic expression with 1 or more terms, separated by +/-, and the terms are written in decreasing order of powers.

The degree of a polynomial: is the degree of the term with the highest degree.
Example: $3x^2y^2 + 4xy^2$ has degree 4

Simplifying an algebraic expression means representing it using as few terms as possible (collecting like terms)

The **Zero** of a polynomial is the value of the variable which makes the polynomial equal to zero

Unit 2

L 1 Monomial Basics

Refer to first half of the Handout: "Algebraic Expressions", for definitions.

A **MONOMIAL** is the product of a variable with a positive integer exponent and real number.

LIKE TERMS are terms with identical variables and identical exponents (not coefficients)

The **DEGREE** of a monomial is the sum of all its exponents.

$\frac{1}{b^5}$ $\frac{1}{2}y$ $3x$ 7 $\sqrt{5a}$ $12a^{\frac{1}{2}}$ $-22a^5b^7$ $2y^{-5}$

Ex 1: Monomial Vs Not a Monomial

Ex 2: Are the following pairs like terms?

1) 2a , -2a
2) 4b , 6ba
3) $3x , -7x^2$
4) abc , -abc
5) $3b^0 , 5$
6) $6x , \frac{4}{x}$
7) $3x^2y , 4xy^2$
8) $11st^2u^3 , 9u^3t^2s$
9) $\frac{2}{5}$, -8
10) 2a , 3ab
11) $3x , 3x^0$
12) $2ax^2 , ax$
13) $2a^2x^3 , -2a^2x^3$
14) Is $2x^{-1}$ a monomial?

Ex 3: Determine the degree of each monomial

Monomial	$5x^2$	$3y^{12}$	-7	$6xy^4$	$3a^3b^3$
Degree					

Example $6xy^4$

$6(\ )(\ )^4 = ?$

If x=3; y=2

$6(3)(2)^4$

$= 6(3)(16)$

$= 288$

We can use Algi-tiles to represent single variable polynomials: Introducing the Tiles .....

+1	+1 Tile	-1	-1 Tile
x	+x Bar	-x	-x Bar
x^2	$+x^2$ Square	$-x^2$	$-x^2$ Square

Note that 2 opposites of the same type cancel each other out when added.

Practice:

2.1 #1-3

Unit 2

L 2 Monomial Operations

- **Adding/ Subtracting**:

Only like terms can be +/- (simplified to a single term)

  – Non like terms cannot be simplified to a single term
  – When you +/- terms, do it to the **coefficients** only.

- **Multiplying/ Dividing**:

They don't have to be like terms.

- **Multiplying:** $(ax^m)(bx^n) = abx^{m+n}$
- **Dividing:** $\frac{ax^m}{bx^n} = \frac{a}{b}x^{m-n}$

Ex 1: Simplify the following monomials

$3x^2 + 4x^2$ _____ Adding

$5x^3y^2 - 3x^3y^2$ _____

$(2a)(5b)$ _____ Multiplying

$4(1.5a)$ _____

$\frac{12x^3y^4}{6x^2y^2}$ _____ Dividing

Practice:
Handout 2.2
2.2 # 1-3
2.3 # 1-5

2.2 Monomial Operations

Remember:

1) Only like terms can be added or Simplified to a single term.
2) Non like terms cannot be simplified to a single term
3) When you add or simplify polynomials do it to the coefficients only
4) When multiplying monomials: $\boxed{ax^m.bx^n = abx^{m+n}}$
5) When dividing monomials: $\boxed{\frac{ax^m}{bx^n} = \frac{a}{b}x^{m-n}}$

Exercise:

1] $x^2 + x^2 =$

2] $2x^3 + 3x^3 =$

3] $x^{-4} + x^4 =$

4] $2x + 4x =$

5] $14x + 7x^2 =$

6] $-6x + 3x =$

7] $-11x - -2x =$

8] $5x^2 - 3x^2 =$

9] $5x^2 - -2x^2 =$

10] $\frac{1}{2}x + \frac{1}{2}x =$

11] $\frac{2}{5}x + \frac{3x}{5} =$

12] $0.37y^2 + 0.11y^2 =$

13] $-2 \times 3 =$

14] $-2a \times 3 =$

15] $(-2a)(3a) =$

16] $(4x)(4x) =$

17] $(7y)(3xy) =$

18] $(-3ab)(4a^2bc) =$

19] $4a(-5a) =$

20] $3c(2d) =$

21] $-2(4xy) =$

22] $-(-2a) =$

23] $\frac{6x^2y}{3xy} =$

24] $\frac{16x^2}{24x^2} =$

25] $\frac{12x^4}{3x} =$

26] $\frac{10x^2}{5x^3} =$ NOT a MONOMIAL

Unit 2

L3 Polynomial Basics

Refer to second half of the Handout: "Algebraic Expressions", for definitions.

A **POLYNOMIAL** is the sum or difference of many unlike MONOMIALS.

Write the terms in decreasing order of degrees.

Ex: $12x^7 + 6x^4 - 7x^2 + 7$

- Evaluating a polynomial consists of finding the numerical value of the polynomial for a given value of the variable.

  Ex.: A stone is thrown from the top of a 25 m high cliff (compared to sea level).

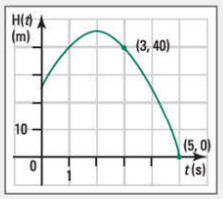
  The polynomial $H(t)$ (in metres) gives the stone's height for a given time t (in seconds).

  $H(t) = -5t^2 + 20t + 25$

  To determine the stone's height after $t = 3$ seconds, evaluate the polynomial for $t = 3$.

  $H(3) = -5 \times 3^2 + 20 \times 3 + 25 = 40$

  The stone is at a height of 40 m at $t = 3$ seconds.



- The zero of a polynomial is any value of the variable which makes the polynomial equal to zero.

  Since $H(5) = 0$, we conclude that 5 is a zero of the polynomial.

  Thus, after $t = 5$ seconds, the stone enters the water.

P(x) is just the notation to name the polynomial

Ex 1: Simplify:

$P(x) = 2x^2 + 5x^3 + 3x + 6 + 3x + 4x^2 + 7 - 5x^3$

Ex 2: Evaluate the above trinomial for x = 2

(ie. Evaluate P(2))

P(2) =

Ex 3: Rewrite each polynomial and give its degree.

a) $4xy^2 + 3x^2y^2$ b) $2 - 5y^2 + 6y$

Ex 4: If $P(x,y) = -3x^2y + 2xy^2 - 2x + 3y - 5$; evaluate

P(-2,1)

Ex 5: A mother is 5 times as old as her daughter.

a) If the girl is x years old, how old is the mother?

b) How old will each be in 13 years?

	Mother	Girl
Now		
In 13 years		

c) What will their total age be in 13 years?

Practice:

2.4 # 1,2

Unit 2

L4 Addition and subtraction of Polynomials

Adding Polynomials: group like terms

Ex 1: $2x^2 - 3x - 3$

$+ \quad -2x^2 + 2x + 6$

$=$

1

Ex 2: Simplify

$3x^2 + (5x + 10x) =$ _____

$(8xy^2 + 9x^2y) + 5xy^2 =$ _____

$(6a + 12b) + (7a + 5b) =$ _____

$(7y + 6) + (8y^2 + 10) =$ _____

2

Subtracting polynomials: subtract each like term.
(It is like adding the opposite of each term)

Ex 3: $2a^2 + 5a + 8$

$- \quad a^2 - 4a + 5$

$=$

Same as: $2a^2 + 5a + 8$

$+ \; -a^2 + 4a - 5$

$=$

3

Ex 4: Simplify by subtracting the polynomials

$7x + (5x + 10x) =$ _____

$8x^2 + 4x + (6x^2 + 2x) =$ _____

$4a + 7b + (12a - 5b) =$ _____

$7c + 6c^2 + (8c^2 - 10) =$ _____

4

Ex 5: Sam works at a Telephone Company he gets $16/h but pays $35 per paycheque for insurance. Pam works in a restaurant; she gets $10/h and $120 tips per pay period.

a) Write an algebraic expression for each of them.

b) Find the total of their incomes.

5

S(h) = 16h – 35 **P(h) = 10h + 120**

T(h) = 26h + 85

c) Find the difference between their incomes

d) If in one pay period they work 30 hours each, what is their total pay?

6

Ex 6: A mother is 5 times as old as her daughter.

a) If the girl is x years old, how old is the mother?

b) How old will each be in 13 years?

	Mother	Girl
Now		
In 13 years		

c) What will their total age be in 13 years?

7

Ex 7: Mix bag Polynomials review

1) Simplify: $3x^2 + 10x^2 - 6x + 4x$
2) True or false:
   a) A monomial can have a negative exponent.
   b) Like terms are monomials with the same variables raised to the same exponents.
   c) A polynomial has at least two UNLIKE TERMS.
3) Simplify: $3a + 5b - (7a + 9b)$
4) Is $4x^2 - 7x + 10$ a trinomial?
5) Simplify: $3x + 7y - (2x - 6y)$
6) Circle the monomials

$\sqrt{5a} \quad 7a^5b^7 \quad y^{-10} \quad 6 \quad 12x^4$

8

Practice:
2.5 # 1-5

10

Unit 2

L5 Multiplication of Polynomials

Case 1- Monomial times a Polynomial:

Examples: 3(2x + 4) =

Distributive Property

Now Try (a) 3x(5x – 2) =

(b) $-2x^2(3x + 5)$ =

(c) $\frac{2}{3}x^2(6x^2 - 9x + 3)$ =

1

Ex 1: Multiplying Polynomials

$3x(5x^2 + 2x) =$ _____

$-5(2x+1) =$ _____

$-2x^2(3x-3) =$ _____

$\frac{3}{2}x^2(6x^3 - 8x + 4) =$ _____

2

Case 2- Binomial times a Binomial:

$$(2x + 2)(x + 4)$$

Option 1: Use Distributive Property:

2x(x + 4) + 2(x + 4)

= $2x^2 + 8x + 2x + 8$

= $2x^2 + 10x + 8$

4

Option 2: Expand and simplify (FOIL)

$(2x+2)(x+4)$

F - First
O - Outside
I - Inside
L - Last

F O I L

= (2x)(x) + (2x)(4) + (2)(x) + (2)(4)

= $2x^2 + 8x + 2x + 8$

= $2x^2 + 10x + 8$

5

2 choices of methods

(2x + 2)(x + 4)

2x(x + 4) + 2(x + 4)

= $2x^2 + 8x + 2x + 8$

= $2x^2 + 10x + 8$

$(2x+2)(x+4)$

F - First
O - Outside
I - Inside
L - Last

F O I L

= $2x^2 + 8x + 2x + 8$

= $2x^2 + 10x + 8$

6

Ex 2: Expand and simplify (FOIL)

$(x-3)(x+5)$

7

Ex 3: Foil practice

$(x+2)(x+2) =$ _____

$(x-3)^2 =$

_____

$(4x-3)(2x+1) =$ _____

8

Practice:
2.6 # 1-6

9

Ex 4: (2.6 # 7 (b))
(3x + 2)(2x – 3) – (x – 1)(2x + 1)

Ex 5: Find the missing factor

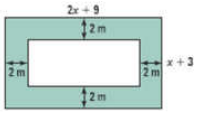
(_____)(2x – 5) = $8x^2 - 20x$

10

Math 3000 pg. 60-61 #14, 18, 20

14. The pool at a municipal park is surrounded by a cement sidewalk as indicated in the figure on the right. The dimensions are in metres.

What is the area of the sidewalk? _____



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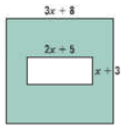
18. Marlene wants to put a rectangular table in the square space of her kitchen. What polynomial represents the space she will have left?

_____



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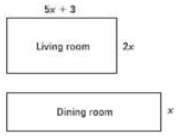
20. The dining room and the living room of a house have the same area. What is the perimeter of the dining room?

_____



15

Practice:
2.6 # 7-10

17

L6 Division of Polynomials

Case 1- Polynomial divided by a constant:

Divide each term by the constant

Ex 1: $(4x + 8) \div 2 =$

Ex 2: $\frac{6x - 18}{3} =$

Ex 3: $\frac{8x^2 + 12x + 16}{4} =$

Case 2- Polynomial divided by a Monomial:

Divide each term by the Monomial. Following the laws of exponents.

Ex 1: $(4x^2 + 2x) \div 2x =$

Ex 2: $\frac{-14x^3 + 35x^2}{7x^2} =$

Ex 3: $\frac{4x^3 + 8x^2 - 6x}{2x} =$

Ex 4: Simplify by dividing

$\frac{9x^3 + 6x^2 - 12x}{3x}$

$\frac{18a^2 + 12ab}{3a}$

$\frac{18x^2 + 8x + 6}{3}$

$\frac{20xy^5 - 15xy^2 + 30x^2y^4}{5xy}$

Practice:

2.7 # 1-6

Unit 2

L7 Greatest common factor

A Factor is an integer that divides evenly into another number.

The factors of 6 are…

The factors of 24 are…

The Greatest Common Factor (GCF) of a polynomial: is the largest factor that divides evenly into each term.

- Factoring is the exact opposite of expanding.
- We expand a product and factor a sum.
- To factor by removing the Greatest Common Factor:
  1. Find the GCF→ the gcf of the coefficients , and the gcf of the variables (for each variable it will be the one with the smallest exponent)
  2. Find the second factor: divide each term in the polynomial by the GCF you found.
  3. Always check by expanding.

Ex 1: Find the gcf

a) 8 , 16 , 40

b) $6x^2$, $24x^3$, $12x^4$

c) $28x^2y^2$, $14x^3y^2$, $21x^2y^3$

d) $15a^6b^7$, $3a^3b^5$, $21a^6b^4$

Ex 2: Factor by removing the gcf

a) $5x + 10y - 15$ b) $12x^2 - 8x$

Ex 3: Factor by pulling out the GCF

$4x+6=$

$9x-15=$

$6x^2+10x=$

$49x^3y^2 - 21x^2y^2 + 14x^3y^3=$

$2a^2b^2-6ab^3+4ab^2=$

$x(x+3) + 2(x+3)=$

Practice:
2.8 # 1-5